

Lecture 18

Recall last day we talked about ‘winding number’ and it is a continuous function from \mathbb{C} to \mathbb{Z} is continuous. Today we will prove two big theorem, **Argument principle** and **Rouche’s theorem**.

Theorem 18.0.1 (Argument Principle) Let $\mathcal{O} \subseteq \mathbb{C}$ open, γ be a closed simple curve such that $\gamma \cup \Sigma\gamma \subseteq \mathcal{O}$. Suppose f is a meromorphic function. Then

$$\frac{1}{2\pi i} \oint_{\gamma} \frac{f'(z)}{f(z)} dz = Z\{\# \text{ zeroes of } f \text{ in } \Sigma\gamma\} - P\{\# \text{ poles of } f \text{ in } \Sigma\gamma\}$$